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Exchange Current Corrections to Neutrino–Nucleus Scattering*

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Abstract

Relativistic exchange current corrections to neutrino–nucleus cross sections are presented assuming non–vanishing strange quark form factors for the constituent nucleons. For charged current processes the exchange current corrections can lower the impulse approximation results by 10% while these corrections are found to be sensitive to both the nuclear density and the strange quark axial form factor of the nucleon for neutral current processes. Implications on the LSND experiment to determine this form factor are discussed.

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It is well known that meson exchange currents play an essential role in realistic descriptions of electroweak interactions in nuclei [1]. For example in the electromagnetic sector, exchange current corrections required by gauge invariance are found to be important in explaining the renormalization of orbital g–factors [2], the threshold radiative neutron capture rates [3], or its inverse, the deuteron photo-disintegration cross section [4] and the transverse (e, e') response functions in the dip region [5]. In addition, shell model studies of first forbidden β-decay rates covering a wide range of nuclei [6] indicate a substantial exchange current contribution to the renormalization of weak axial charge in medium, as predicted by Kubodera, Delorme and Rho in 1978 [7]. Thus, empirical evidences abound suggesting that both electromagnetic and weak axial currents are subject to renormalizations in nuclei due to exchange currents. It is therefore interesting to examine the effects of exchange currents, if any, in neutrino–nucleus scattering where both vector and axial currents are involved simultaneously.

Another reason to investigate exchange current corrections to neutrino–nucleus scattering is that it has been receiving increasing attention as a means to determine the strangeness matrix elements of the nucleon [8–14]. The measurement of polarized structure function g_1 and the extraction of the sum rule indicated the possibility of a rather large strange quark axial matrix element for the proton [15], and has inspired numerous works attempting to understand the role of hidden flavor in nucleons. However, the situation regarding the strangeness degrees of freedom in the nucleon is far from clear and it is hoped that neutrino-nucleus interactions might be able to shed a new light into this problem [16]. In order to extract strangeness matrix elements for the nucleon from neutrino–nucleus scattering it is necessary to reliably calculate the cross sections assuming finite strange quark form factors [17]. The kinematics of neutrino–nucleus interactions involved in determining the strange content of the nucleon ranges from low-energy inelastic scattering [10] to the quasi-elastic region [12]. Experience from electron scattering suggests that exchange current corrections to cross sections in this kinematic range might be important.

In this letter two-body exchange current corrections to the impulse approximation in

low and intermediate energy neutrino–nucleus scattering are presented using the generalization of a method developed by Chemtob and Rho [18]. As shown below this method is powerful enough to estimate exchange current corrections to both neutral and charged current processes simultaneously assuming finite strange quark form factors of the nucleon. In addition, the formalism involved in this approach is model independent in the sense that no nucleon–nucleon interaction need to be specified. As examples, relativistic two-body exchange current corrections to neutral and charged current neutrino–nucleus cross sections are evaluated for several nuclear densities assuming nuclear matter and using the kinematics of the ongoing LSND experiment to measure the strange axial form factor of the nucleon [12].

It is convenient to write the neutral and charged currents of a *free* nucleon, $J_\mu^{Z^0}$ and $J_\mu^{W^\pm}$, in terms of $SU(3)$ vector, V_μ^a, and axial, A_μ^a, currents where $a = 0$ for singlet and $a = 1 \rightarrow 8$ for octet currents, respectively.

$$J_\mu^{Z^0} = V_\mu^3 - A_\mu^3 - 2\,\sin^2\theta_W \left(V_\mu^3 + \frac{1}{\sqrt{3}} V_\mu^8 \right) - \frac{1}{2}\left(V_\mu^0 - \frac{2}{\sqrt{3}} V_\mu^8 \right) + \frac{1}{2}\left(A_\mu^0 - \frac{2}{\sqrt{3}} A_\mu^8 \right) \quad (1)$$

$$J_\mu^{W^\pm} = \left[\left(V_\mu^1 \pm i V_\mu^2 \right) - \left(A_\mu^1 \pm i A_\mu^2 \right) \right] \cos\theta_C + \left[\left(V_\mu^4 \pm i V_\mu^5 \right) - \left(A_\mu^4 \pm i A_\mu^5 \right) \right] \sin\theta_C \quad (2)$$

In these definitions of weak neutral and charged currents, θ_W and θ_C are the Weinberg and Cabbibo angles, respectively, and the last two terms in Eq. (1) are usually referred to as the strange quark vector and axial currents of the nucleon. Small QED, QCD and heavy quark corrections to $J_\mu^{Z^0}$ [9] as well as contributions from the charmed quarks to $J_\mu^{W^\pm}$ are ignored. Thus, the problem addressed in this letter is to estimate exchange current corrections to the above currents when the nucleon is immersed in nuclear medium. Other medium effects, such as density dependent off–shell form factors and effective nucleon and meson masses, are not considered in this work in order to clearly isolate possible exchange current effects in many body systems.

The starting point is to assume the chiral–filtering conjecture [7] which states that the dominant exchange current contribution in nuclei at low and intermediate energies comes from the exchange of a single pion whose production amplitude is evaluated in the soft–

pion limit. As mentioned, there exists a well–known method by Chemtob and Rho [18] to construct soft–pion exchange current operators which exploits soft–pion theorems and current algebra techniques pioneered by Adler [19]. There are several advantages of using this approach in solving the problem at hand. First, the method of Chemtob and Rho has been applied in the past to various low and intermediate energy phenomena and proved to be a reliable technique for estimating exchange current corrections [1]. For example in their analysis of first forbidden β–decay transitions, Warburton *et al.* find that exchange current corrections to the axial charge may reliably be estimated in the the soft–pion exchange dominance approximation [6]. Furthermore, most of the discrepancy between the measured deuteron photo–disintegration rates involving small energy and *large* momentum transfers [20] and its impulse approximation prediction can be explained by soft–pion exchange corrections [4]. This came as a surprise since the soft–pion dominance approximation was thought to be applicable only to processes involving small momentum transfers as in first forbidden β–decay transitions [21].

A justification for the success of the soft–pion exchange dominance for finite momentum transfers was proposed by Rho who based his arguments on Weinberg's derivation of nuclear forces from chiral Lagrangians [22]. Using chiral power counting Rho has argued that to the leading order, *i.e.* at the tree level, the short range part of two–body meson exchange current corresponding to a nuclear force predicted by a given chiral Lagrangian is considerably suppressed. Thus the dominant contribution to two–body currents comes from the long ranged part represented by the soft–pion exchange. Consequently, Park, Towner and Kubodera [23] have calculated corrections to the axial charge exchange current operators beyond the the soft–pion dominance approximation using heavy–fermion chiral perturbation theory. They find that loop corrections to the soft–pion exchange current operators is of the order of 10%, and argued that their results are consistent with the claims of Warburton *et al.* and support the chiral–filtering conjecture. Thus, the soft–pion technique of constructing exchange current operators seems to be a plausible approximation in both electromagnetic and weak axial sectors, and it is natural to apply this method to neutrino–nucleus scatter-

ing where both vector and axial currents are involved. Furthermore, there are theoretical arguments suggesting that corrections to the soft–pion dominance approximation are small, even for finite momentum transfers extending to the quasi–elastic region [21,22].

Finally, the main advantage of using the method of Chemtob and Rho is the equal treatment of all the vector and axial currents entering in neutral and charged currents as described below. The quantity of interest in this method is the amplitude for pseudoscalar meson production off a nucleon by an external current, denoted by $\langle N(p')\phi^b(q)|J^a_\mu(k)|N(p)\rangle$. Here k and q are the four momenta of the probing current J^a_μ, and the produced meson ϕ^b, respectively, while a and b are $SU(3)$ indices ($a = 0 \to 8$ while $b = 1,2$ or 3 for pion production). In the present case J^a_μ may be any one of the $SU(3)$ vector or axial currents appearing in Eqs. (1) and (2), and the meson production amplitude is evaluated in the soft–meson limit of $q \to 0$. This soft–meson production amplitude, derived by Adler [19] and used by Chemtob and Rho [18], may be written in the generalized form as

$$\lim_{q\to 0}\langle N(p')\phi^b(q)|J^a_\mu(k)|N(p)\rangle = \frac{i}{F_\phi}\int d^4x \lim_{q\to 0}(-iq^\nu)\langle N(p')|T\left(A^b_\nu(x)J^a_\mu(0)\right)|N(p)\rangle$$
$$-\frac{i}{F_\phi}\langle N(p')|\left[Q^b_5(x), J^a_\mu(0)\right]_{x_0=0}|N(p)\rangle \qquad (3)$$

Here $Q^a_5(x) \equiv \int d^3x\, A^a_0(x)$ is the axial charge and F_ϕ is the decay constant for the pseudoscalar meson ϕ. As shown in [24], the only contributions to the first term in the soft–meson limit come from pole terms where the matrix element $\langle N(p')|T\left(A^b_\nu(x)J^a_\mu(0)\right)|N(p)\rangle$ behaves like $1/q_\mu$. The second term may be simplified by using the well–known $SU(3)\otimes SU(3)$ current algebra

$$\left[Q^a_5(x), V^b_\mu(0)\right]_{x_0=0} = if_{abc}A^c_\mu(0) \qquad \left[Q^a_5(x), A^b_\mu(0)\right]_{x_0=0} = if_{abc}V^c_\mu(0) \qquad (4)$$

and has no contributions from singlet currents unlike in the first term where both $SU(3)$ singlet and octet currents can contribute. Since J^a_μ can be any of the $SU(3)$ vector or axial currents, Eq. (3) may be applied to *all* the components of weak neutral and charged currents in Eqs. (1) and (2) simultaneously. Thus, it is the use of current algebra in Eq. (3), which rotates around the vector and axial octet currents, that makes this method particularly suit-

able to estimate exchange current corrections in neutrino scattering at low and intermediate energies assuming finite strange quark form factors.

Two–body operators for neutral and charged–current induced exchange currents may be constructed in a straightforward manner following [18]. The non–relativistic limit of these operators have previously been used to calculate exchange current corrections to neutrino–deuteron scattering in the original $SU(2)$ version supplemented by finite q corrections [25]. In the present application, the operators are fully relativistic and the method is generalized to $SU(3)$ to accomodate finite strange quark form factors. The conservation of the vector current has been checked both analytically and numerically using the prescription outlined in [19]. Figure 1a shows differential cross sections for the neutral current reaction $^{12}C(\nu, \nu' p)$ plotted against the kinetic energy of the ejected nucleon T_F. The calculation was performed using the relativistic Fermi Gas model formalism as in Horowitz *et al.* with zero binding energy. Furthermore, to simulate the LSND experiment, the nucleons are assumed to be ejected quasi-elastically from the target nuclei by neutrinos with a beam energy of 200 MeV, and only $1p1h$ final states are considered when taking matrix elements of two–body operators since the phase space for $2p2h$ final states should be highly suppressed for the LSND kinematics [12].

The differential cross sections are parameterized by the strange quark magnetic, $F_2^s \equiv F_2^s(Q^2 = 0)$, and axial, $G_A^s \equiv G_A^s(Q^2 = 0)$, form factors of the nucleon. The $Q^2 \equiv -k^2$ dependence for F_2^s and G_A^s are assumed to be [12]

$$F_2^s(Q^2) \equiv F_2^0(Q^2) - \frac{2}{\sqrt{3}} F_2^8(Q^2) \quad G_A^s(Q^2) \equiv G_A^0(Q^2) - \frac{2}{\sqrt{3}} G_A^8(Q^2) \tag{5}$$

where

$$F_2^{0,8}(Q^2) \equiv \frac{F_2^{0,8}(0)}{(1+\frac{Q^2}{4M_N^2})(1+\frac{Q^2}{M_V^2})^2} \quad G_A^{0,8}(Q^2) \equiv \frac{G_A^{0,8}(0)}{(1+\frac{Q^2}{M_A^2})^2} \tag{6}$$

In these definitions M_N is the nucleon mass and the vector and axial masses are set to M_V = 840 MeV and M_A = 1030 MeV, respectively. The octet form factors at $Q^2 = 0$ are known quantities given by $F_2^8(0) \equiv \sqrt{3}/2(\kappa_p + \kappa_n)$ and $G_A^8(0) \equiv \sqrt{3}/6(3F - D)$, while the unknown

singlet form factors $F_2^0(0)$ and $G_A^0(0)$ determine the strange quark form factors F_2^s and G_A^s which are assumed to be $F_2^s = -0.21$ and $G_A^s = -0.19$ for this work. The results obtained by assuming no strange form factors ($F_2^s = G_A^s = 0$) are qualitatively similar but there is an overall 20% reduction in the differential cross sections. The exchange current corrections are found to be sensitive to the Fermi momentum k_F of the model. For $k_F \approx 200$ MeV there are cancellations between the vector and axial contributions to the exchange current correction leading to little change from the impulse approximation results. However, for $k_F \approx 300$ MeV and beyond there are considerable corrections to the impulse approximation from exchange currents as shown in the figure.

Figure 1b shows the cross sections for the inclusive charged current process $^{12}C(\nu_{\mu^-}, \mu^-)X$ for several nuclear densities obtained by folding the LSND neutrino energy distribution [26]. Here the effect of exchange current corrections varies from 5 to 10% as the Fermi momentum is increased from 200 to 300 MeV. For $k_F = 225$ MeV, which is the usual value used for ^{12}C, the total cross–section is reduced from 24 to 22.7 ($\times 10^{-40}\mathrm{cm}^2$). This reduction is not enough to explain the recently measured value reported by the LSND collaboration of $(8.3 \pm 0.7$ stat. $\pm$ 1.6 syst.$) \times 10^{-40}\mathrm{cm}^2$ [26].

Another application of the present work is the prediction of the proton–to–neutron quasi–elastic yield $R(p/n) \equiv \sigma(\nu, \nu' p)/\sigma(\nu, \nu' n)$ which is currently being measured at the LSND $^{12}C(\nu, \nu' N)$ experiment. In this experiment $R(p/n)$ is integrated over T_F and the results are plotted as functions of G_A^s for several values of F_2^s. This has been done in Figures 2a and 2b where results for both neutrino and anti–neutrino scattering are shown assuming $k_F =$ 225 MeV. It is important to note that for kinematical reasons the LSND experiment limits the range of integration between $50 \leq T_F \leq 120$ MeV [12]. Because of this cut–off imposed by the experiment, all modifications due to exchange currents for $T_F \leq 50$ MeV are ignored and as a result there is only about 5% change from the impulse approximation results in the ratio for the neutrino scattering while this change is about 15% for anti–neutrinos.

To conclude, relativistic exchange current corrections have been applied to neutrino–nucleus scattering assuming finite strange quark form factors of the nucleon. The gener-

alized version of the method of Chemtob and Rho used in this work is so far the most economical way to estimate exchange current corrections to low and intermediate energy neutrino–nucleus scattering since it treats all the $SU(3)$ vector and axial currents on the same footing. As examples, soft–pion exchange current corrections have been applied to quasi–elastic neutrino–nucleus scattering using a simple Fermi Gas model and kinematics of the on–going LSND experiment. The differential cross sections for the $^{12}C(\nu, \nu' p)$ reaction is found to be sensitive to the values of the strange quark form factors while the exchange current corrections to the cross section were found to become more important with increasing nuclear density. However, because of an experimental kinematical cut, these exchange current effects are considerably reduced when evaluating the integrated ratio of proton–to–neutron yields currently being measured at LSND. For the charged current case exchange current effects reduce the impulse approximation results by 5 to 10% depending on the nuclear density. Nevertheless, the discrepancy between theory and experiment for the recently reported $^{12}C(\nu_{\mu^-}, \mu^-)X$ total cross section remains unexplained. Finally, an extension of the present application to finite nuclei is in progress.

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FIGURE CAPTIONS

FIG 1. a) ${}^{12}C(\nu,\nu' p)$ differential cross section versus the kinetic energy of the ejected nucleon, T_F for several values of Fermi momentum k_F. The incident neutrino energy is 200 MeV and the values for the strange quark form factors are $F_2^s = -0.21$ and $G_A^s = -0.19$. The long dashed curve is the impulse approximation result while the solid curves have been obtained with the full exchange current corrections. Results may be identified by their values at the quasi–elastic peak around $T_F = 30$ MeV. For $k_F = 200$ MeV, there is almost no difference between the two results and the values at the quasi–elastic peak are both about $70 \times 10^{-42}\mathrm{cm}^2/\mathrm{MeV}$. At the quasi–elastic peak the impulse approximation result for $k_F = 300$ MeV has the value of $60 \times 10^{-42}\mathrm{cm}^2/\mathrm{MeV}$ while the corresponding value with exchange current corrections is $50 \times 10^{-42}\mathrm{cm}^2/\mathrm{MeV}$. b) ${}^{12}C(\nu_{\mu^-},\mu^- p)X$ total cross section obtained by folding the LSND neutrino flux [26] versus k_F. The long dashed curve is the impulse approximation result while the solid curve is obtained with the full exchange current corrections.

FIG. 2 a) Ratios of integrated proton–to–neutron quasi–elastic yield for the ${}^{12}C(\nu,\nu' N)$ reaction as functions of G_A^s for two values of strange magnetic form factor F_2^s. In each case, the dashed line is the impulse approximation result while the solid line has been corrected for meson exchange currents. The incident neutrino energy is assumed to be 200 MeV for both cases and the range of integration was chosen to be $50 \leq T_F \leq 120$ MeV to simulate the LSND experiment [12]. b) Same as in a) but for anti–neutrino scattering.